Revisiting the semiclassical gravity scenario for gravitational collapse

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Abstract.
The existence of extremely dark and compact astronomical bodies is by now a well established observational fact. On the other hand, classical General Relativity predicts the existence of black holes which fit very well with the observations, but do lead to important conceptual problems. In this contribution we ask ourselves the straightforward question: *Are the dark and compact objects that we have observational evidence for black holes in the sense of General Relativity?* By revising the semiclassical scenario of stellar collapse we find out that as the result of a collapse some alternative objects could be formed which might supplant black holes.

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INTRODUCTION

An already impressive quantity of high-quality observational data points towards the existence of very dark and compact astronomical bodies. On the theoretical side, classical General Relativity (GR) predicts under quite general and reasonable conditions (some positive-energy condition applied to the matter stress-energy-momentum tensor) that stars of large masses (greater than about 3 solar masses) cannot avoid complete collapse once they have exhausted their nuclear fuel. When the collapsing star reaches a radius smaller than its gravitational radius (*i.e.*, its Schwarzschild radius $r_g = GM/c^2$ if it is non-rotating) there is a region of spacetime that cuts itself off forever from the rest of the universe; nothing, not even light can escape from this region. A black hole in GR is defined as precisely this cut-off region. The surface that separates this region from the rest of the universe is called the black-hole event horizon.

The notion of black hole fits so nicely with the observations that the extremely compact and dark objects found out there in the sky are customarily called black holes or more gingerly black hole candidates. But are they really black holes in the precise sense of classical GR? At the same time that classical GR predicts the existence of black holes it also predicts that inside the event horizon of a standard (non-extremal) black hole there will always be a singularity. The formation of singularities is a worrisome feature of classical GR that to many people suggests one has to unavoidably incorporate quantum ingredients into the study of gravity.

One of the most conservative ways of incorporating quantum aspects into GR is to treat the geometry as classical but the matter as quantum, with the expectation value of

the stress-energy-momentum tensor (SET) in the specific quantum state of matter acting as the source of gravity. This framework constitutes the so-called semiclassical GR. It is standard to write the semiclassical Einstein equations as

$$G_{\mu\nu} = 8\pi \left(T^{c}_{\mu\nu} + T^{Q}_{\mu\nu} \right) , \qquad \text{with} \qquad T^{Q}_{\mu\nu} \equiv \langle \psi | \widehat{T}_{\mu\nu} | \psi \rangle . \tag{1}$$

Here, one separates the expectation value of the total SET into a classical part, $T^{c}_{\mu\nu}$, and a purely quantum contribution calculated through renormalization. Significant deviations from the standard Einstein equations can appear only if the renormalized SET in equation (1) becomes comparable with the classical SET. Ideally, in this approach one should self-consistently solve the semiclassical Einstein equations, but this is in general rather complicated. Instead, what is typically done is to take a fixed geometry, calculate the renormalized stress-energy-momentum tensor (RSET) of the quantum fields in specific states and analyze what the effects of this RSET, taken as an additional source of gravity, would be.

Within semiclassical GR it was found that black holes cannot be stationary [1] but that they must evaporate [2]. Conceptually, these evaporating black holes might be very different from classical black holes, in the sense that they might have no strict event horizon after all [3]. However, the evaporation of a stellar-mass black hole is so slow that in astrophysical terms the latter would be completely indistinguishable from a classical black hole. Instead of a strict event horizon it would possess a long-living trapping horizon. A light signal might escape from the evaporating black hole, but only after the evaporation process has developed sufficiently, which will take many times the present age of the universe.

In this contribution we want to take one step back and ask ourselves whether semiclassical GR leads unavoidably to the formation of evaporating black holes or, in other words, whether it can provide qan alternative route producing dark and compact stellar-mass objects having, however, no horizons of any kind. These objects might in principle be suitable for astrophysical exploration (either detection or elimination) in a reasonable time frame.

THE RESULTS

Let us revisit the semiclassical GR scenario for stellar collapse. Consider a star of mass M in hydrostatic equilibrium in empty space. For such a configuration the appropriate quantum state is well known to be the Boulware vacuum state $|0_{\rm B}\rangle$ [1], which is defined unambiguously as the state with zero particle content for static observers. This quantum state is regular everywhere both inside and outside the star (this state is also known as the static, or Schwarzschild, vacuum [4]). If the star is sufficiently dilute (so that its radius is very large compared to $2M$), then the spacetime is nearly Minkowskian and such a state will be virtually indistinguishable from the Minkowski vacuum. Hence, the expectation value of the RSET will be negligible throughout the entire spacetime. This is the reason why, when calculating the spacetime geometry associated with a dilute star, one only needs to care about the classical contribution to the SET.

Imagine now that, at some moment, the star begins to collapse. The evolution proceeds as in classical general relativity, but with some extra contributions as spacetime dynamics will also affect the behaviour of any quantum fields that are present, giving place to *both* particle production *and* additional vacuum polarization effects. Contingent upon the standard scenario being correct, if we work in the Heisenberg picture there is a single globally defined regular quantum state $|C\rangle = |\text{collapse}\rangle$ that describes these phenomena.

If the collapse proceeds very quickly, with the entire structure almost freely falling into itself, then it has (numerically) been shown (see for example [5]) that the RSET maintains negligible values throughout the collapse, well beyond the moment at which the trapping horizon forms. One can neglect these quantum effects insofar as the dynamics of the collapse is concerned, at least up to the appearance of a trapped region. Only when the collapse generates large curvatures, that is, near the incipient singularity inside the trapped region, do quantum effects become important. The only residual quantum effect that can in principle be observable from outside is the presence of Hawking radiation coming out from the trapping horizon, and the consequent shrinking of the trapped region. Remember nonetheless that both these effects are almost unnoticeable in an astrophysical context.

This constitutes the standard view of the collapse process in semiclassical GR. However, we found [6] that if for whatever reason the collapse deviates significantly from free fall, then the RSET can contribute appreciably to the dynamics of collapse itself, *before* any trapped region has formed. In these situations the RSET provides an energy-condition violating contribution that will tend to slow down further the collapse. This opens up the possibility that vacuum polarization effects, encoded in the RSET, delay indefinitely the appearance of a trapped region by completely halting the collapse, or by producing an asymptotic approach to trapping horizon formation. In the latter case we also showed [7] that the resulting object might maintain most of the thermodynamic properties of black holes, as it could evaporate by emitting a Planckian spectrum of particles with an associated temperature indistinguishable in practice from the Hawking temperature.

If this was finally the route taken by nature, the new quantum-corrected bodies that substitute classical black holes would not be evaporating black holes, as the standard view maintains, but "black stars" — objects radically different from black holes in several respects. They would be material bodies, with a real and observable (although extremely red-shifted) surface and a non-empty interior, filled with matter at a density at least one order of magnitude greater than that of a neutron star. Having no horizons, they would not put any fundamental barrier to their complete astrophysical exploration. They would be supported by the most basic form of quantum pressure, the one provided by vacuum polarization itself.

The existence of this huge vacuum polarization energy would be tied up to the fact of having a configuration maintaining itself close to horizon formation, and not to the specific values of the curvature in this same region. In the case that they Hawking-like evaporate, (we don't know whether or not this might require some fine-tuning), the internal structure of these bodies would be that of a rainbow of temperature: Imagining the body as composed of thin spherical shells, each of these will be slowly shrinking, chasing but never reaching its own horizon; in this way to each shell one associates a

Hawking-like temperature which steadily increases towards the centre.

THE CALCULATION

For simplicity, consider a massless quantum field and restrict the analysis to spherically symmetric solutions. Every mode of the field can (neglecting back-scattering) be described as a wave coming in from $\mathscr{I}^-$ (*i.e.*, from $r \to +\infty$, $t \to -\infty$), going inwards through the star till bouncing at its center ($r = 0$), and then moving outwards to finally reach $\mathscr{I}^+$. As in this paper we are going to work in $1+1$ dimensions (*i.e.*, we shall ignore any angular dependence), for convenience instead of considering wave reflections at $r = 0$ we will take two mirror-symmetric copies of the spacetime of the collapsing star glued together at $r = 0$. In one copy r will run from $-\infty$ to 0, and in the other from 0 to $+\infty$. Then one can concentrate on how the modes change on their way from $\mathscr{I}^-_{\text{left}}$ (*i.e.*, $r \to -\infty$, $t \to -\infty$) to $\mathscr{I}^+_{\text{right}}$ (*i.e.*, $r \to +\infty$, $t \to +\infty$). Hereafter, we will always implicitly assume this construction and will not explicitly specify "left" and "right" except where it might cause confusion.

With reference to this construction, we shall start by considering a set of affine coordinates U and W, defined on $\mathscr{I}^-_{\text{left}}$ and $\mathscr{I}^-_{\text{right}}$ respectively. These coordinates are globally defined over the spacetime and the metric can be written as

$$g = -C(U,W)\,\mathrm{d}U\,\mathrm{d}W \ . \tag{2}$$

Given that we shall be concerned with events which lie outside of the collapsing star on the right-hand side of our diagram, we can also choose a second double-null coordinate patch (u,W), where u is taken to be affine on $\mathscr{I}^+_{\text{right}}$, in terms of which the metric is

$$g = -\bar{C}(u,W)\,\mathrm{d}u\,\mathrm{d}W \ , \qquad \text{with} \qquad C(U,W) = \bar{C}(u,W)/\dot{p}(u) \ . \tag{3}$$

Of course $U = p(u)$ describes the coordinate transformation which in turn is the red-shift function, and encodes how a wave packet of initial frequency ω' (on $\mathscr{I}^-_{\text{left}}$) gets red-shifted in passing in, through, and out the collapsing star: on $\mathscr{I}^+_{\text{right}}$, $\omega(u,\omega') = \dot{p}(u)\omega'$. Furthermore, as long as we are outside the collapsing star it is safe to assume that a Birkhoff-like result holds, and take $\bar{C}(u,W)$ as being that of a static spacetime.

Now for *any* massless quantum field, the RSET (corresponding to a quantum state that is initially Boulware) has components [4, 8]

$$T^Q_{UU} \propto C^{1/2}\,\partial^2_U\,C^{-1/2} \ ; \qquad T^Q_{WW} \propto C^{1/2}\,\partial^2_W\,C^{-1/2} \ ; \qquad T^Q_{UW} \propto R \ . \tag{4}$$

The coefficients arising here are not particularly important, and will in any case depend on the specific type of quantum field under consideration.

The components T^Q_{WW} and T^Q_{UW} will necessarily be well behaved throughout the region of interest; in particular they are the same as in a static spacetime and are known to be regular. On the contrary T^Q_{UU} shows a more complex structure due to the non-trivial relation between U and u. A brief computation yields

$$C^{1/2}\,\partial^2_U\,C^{-1/2} = \frac{1}{\dot{p}^2}\left[\bar{C}^{1/2}\,\partial^2_u\,\bar{C}^{-1/2} - \dot{p}^{1/2}\,\partial^2_u\,\dot{p}^{-1/2}\right] . \tag{5}$$

The key point here is that we have two terms, one ($\bar{C}^{1/2}\,\partial_u^2\,\bar{C}^{-1/2}$) arising purely from the static spacetime outside the collapsing star, and the other ($\dot{p}^{1/2}\,\partial_u^2\,\dot{p}^{-1/2}$) arising purely from the dynamics of the collapse. If, and only if, the horizon is assumed to form at finite time will the leading contributions of these two terms cancel against each other — this is the standard scenario.

Indeed the first term is exactly what one would compute from using standard Boulware vacuum for a static star. As the surface of the star recedes, more and more of the static spacetime is "uncovered", and one begins to see regions of the spacetime where the Boulware contribution to the RSET is more and more negative, in fact diverging as the surface of the star crosses the horizon.

To study the regular or divergent behaviour of the RSET in different geometries it is convenient to use a set of regular coordinates. Let us choose Painlevé-Gullstrand coordinates in which the metric is expressed as

$$ds^2 = -\left[c^2 - v^2(t,x)\right]\mathrm{d}t^2 - 2\,v(t,x)\,\mathrm{d}t\,\mathrm{d}x + \mathrm{d}x^2\ . \tag{6}$$

Taking the convention $v<0$, in these coordinates the trapping horizon is located at the point at which $c+v=0$. Then, the components of the RSET which can be potentially divergent are T^Q_{tx} and T^Q_{xx}. Explicitly, the terms that can be divergent are

$$T^Q_{tx} = -\frac{\dot{p}^2}{c+v}\,T^Q_{UU} + \cdots \tag{7}$$

$$T^Q_{xx} = \frac{\dot{p}^2}{(c+v)^2}\,T^Q_{UU} + \cdots \tag{8}$$

Calculation assuming normal horizon formation

Hereafter, we shall for simplicity restrict our attention to the case $c(x)\equiv 1$. Placing the horizon at $x=0$ for convenience, we can write the asymptotic expansion

$$v(x) = -1 + \kappa\,x + \kappa_2\,x^2 + \cdots\ , \tag{9}$$

where κ can be identified with the surface gravity [7, 9].

Consider first the static Boulware term in equation (5). We have (placing the horizon at $x=0$ for convenience)

$$\bar{C} = -\frac{\dot{p}}{U_x\,W_x} = -\frac{1}{u_x\,W_x} = 1 - v(x)^2 \approx 2\,\kappa\,x\ . \tag{10}$$

The relevant derivative in ∂_u is then that with respect to x, and we can write

$$\bar{C}^{1/2}\,\partial_u^2\,\bar{C}^{-1/2} \approx (2\,\kappa\,x)^{1/2}\,\kappa\,x\,\partial_x\left(\kappa\,x\,\partial_x(2\,\kappa\,x)^{-1/2}\right) = \kappa^2/4\ . \tag{11}$$

In fact, keeping the sub-leading terms one finds

$$\bar{C}^{1/2}\,\partial_u^2\,\bar{C}^{-1/2} = \frac{\kappa^2}{4} + \mathscr{O}(x^2). \tag{12}$$

By equations (7) and (8), it is clear that because of the constant term $\kappa^2/4$, the components T_{tx} and T_{xx} of the RSET contain contributions that diverge as x^{-1} and x^{-2}, respectively, as $x \to 0$. (The sub-leading terms lead to finite contributions of order $\mathcal{O}(x)$ and $\mathcal{O}(1)$ respectively.)

In counterpoint, assuming horizon formation, let us now calculate the dynamical contribution to the RSET ($\dot{p}^{1/2}\,\partial_u^2\,\dot{p}^{-1/2}$). It is well known that any configuration that produces a horizon at a finite time $t_{\rm H}$ leads to an asymptotic (large u) form

$$p(u) \approx U_{\rm H} - A_1\, {\rm e}^{-\kappa u}\,, \tag{13}$$

where U_H and A_1 are suitable constants. Taking into account the asymptotic expression (9) for $v(x)$ near $x = 0$, it is very easy to see that the potential divergence at the horizon due to the static term is exactly cancelled by the dynamical term. In this way we have recovered the standard result that the RSET at the horizon of a collapsing star is regular.

However, the previous relation is an asymptotic one, and for what we are most interested in (the value of the RSET close to horizon formation) it is important to take into account extra terms that will be sub-dominant at late times. Indeed, we can describe the location of the surface of a collapsing star that crosses the horizon at time $t_{\rm H}$ by

$$x = r(t) - 2M = \xi(t) = -\lambda\,(t - t_{\rm H}) + \cdots\,, \tag{14}$$

where the expansion makes sense for small values of $|t - t_{\rm H}|$, and λ represents the velocity with which the surface crosses the gravitational radius. Let t_0 be the time at which a right-moving light ray corresponding to null coordinates u and U crosses the surface of the star. Then on the one hand

$$t_f - t_0 = \int_{\xi(t_0)}^{x_f} \frac{{\rm d}x'}{1 + v(x')}\,, \tag{15}$$

which for $t_0 \approx t_{\rm H}$ (implying $r(t_0) \approx 2M$) leads to the approximate expression for $u := \lim_{t_f \to +\infty}\left(t_f - x_f\right)$

$$u \approx (t_0 - t_{\rm H}) - \frac{1}{\kappa}\ln\left(-\lambda\,(t_0 - t_{\rm H})\right) + C_1\,, \tag{16}$$

so that

$$t_0 - t_{\rm H} \approx C_2\,\frac{{\rm e}^{-\kappa u}}{\lambda} + \cdots \tag{17}$$

On the other hand, since $U(t_0)$ is simply some regular function, we have

$$U(t_0) = U_{\rm H} + U'_{\rm H}\,(t_0 - t_{\rm H}) + \frac{U''_{\rm H}}{2}\,(t_0 - t_{\rm H})^2 + \cdots \tag{18}$$

Inserting (17) into (18) we obtain an asymptotic expansion

$$p(u) = U_{\rm H} - A_1\,{\rm e}^{-\kappa u} + \frac{A_2}{2}\,{\rm e}^{-2\kappa u} + \frac{A_3}{3!}\,{\rm e}^{-3\kappa u} + \cdots\,. \tag{19}$$

Then

$$\dot{p}^{1/2}\,\partial_u^2\,\dot{p}^{-1/2} = \frac{\kappa^2}{4} + \left[-\frac{1}{2}\frac{A_3}{A_1} + \frac{3}{4}\left(\frac{A_2}{A_1}\right)^2\right]\kappa^2\,\mathrm{e}^{-2\kappa u} + \mathscr{O}\left(\mathrm{e}^{-3\kappa u}\right)\,. \tag{20}$$

The point is that this has a universal contribution coming from the surface gravity, plus complicated sub-dominant terms that depend on the details of the collapse. It is important to note, however, that the corresponding additional contributions to the RSET are finite, in contrast to that associated with the first term. Indeed, for small values of x,

$$u \approx t - \frac{1}{\kappa}\ln x + \mathrm{const}\,, \qquad \text{or} \qquad \mathrm{e}^{-\kappa u} \propto x\,\mathrm{e}^{-\kappa t}\,, \tag{21}$$

and so the second term in the right-hand side of equation (20) is $\mathscr{O}(x^2)$, and by equation (8) gives an $\mathscr{O}(1)$ contribution to T_{xx} that does not depend on x, but depends on time as $\mathrm{e}^{-2\kappa t}$. In addition, from a comparison of equations (17)–(19) we see that

$$\frac{A_2}{A_1} \propto \frac{1}{\lambda}\,, \qquad \frac{A_3}{A_1} \propto \frac{1}{\lambda^2}\,, \tag{22}$$

so the leading sub-dominant term in the RSET is inversely proportional to the square of the speed with which the surface of the star crosses its gravitational radius. In particular, at horizon crossing, that is at $t = t_{\mathrm{H}}$, the value of the RSET can be as large as one wants provided one makes λ very small. This would correspond to a very slow collapse in the proximity of the trapping horizon formation. Thus, there is a concrete possibility that (energy condition violating) quantum contributions to the RSET could lead to significant deviations from classical collapse when a trapping horizon is just about to form.

Calculation assuming asymptotic horizon formation

Another interesting case one may want to consider is one in which the horizon is never formed at finite time, but just approached asymptotically as time runs to infinity. In particular, in reference [7] it was shown that collapses characterized by an exponential approach to the horizon,

$$r(t) = 2M + B\mathrm{e}^{-\kappa_{\mathrm{D}} t}\,, \tag{23}$$

lead to a function $p(u)$ of the form

$$p(u) = U_{\mathrm{H}} - A_1\mathrm{e}^{-\kappa_{\mathrm{eff}} u}\,, \tag{24}$$

where κ_{eff} is half the harmonic mean between κ and the rapidity of the exponential approach κ_{D},

$$\kappa_{\mathrm{eff}} = \frac{\kappa\,\kappa_{\mathrm{D}}}{\kappa + \kappa_{\mathrm{D}}}\,, \tag{25}$$

so that one always has $\kappa_{\mathrm{eff}} < \kappa$. In this case, the calculation of the dynamical part of the RSET leads to exactly the same result as when using expression (13), modulo the substitution of κ by κ_{eff}. However, the non-dynamical part of the RSET remains unchanged. This implies that now, at leading order

$$\mathrm{RSET}(x \approx 0) \approx \frac{1}{\kappa^2 x^2}\left(\kappa_{\mathrm{eff}}^2 - \kappa^2\right) = -\frac{\kappa\,(2\,\kappa_{\mathrm{D}} + \kappa)}{(\kappa_{\mathrm{D}} + \kappa)^2\,x^2}\,, \tag{26}$$

which obviously diverges in the limit $x \to 0$. We stress that this result does not contradict the Fulling–Sweeny–Wald theorem [10], as the calculation applies only outside the surface of the star (*i.e.*, for $x \geq \xi(t)$), and so the divergence appears only at the boundary of spacetime. Nevertheless, particularizing to $x = \xi(t)$, this again indicates that there is a concrete possibility that energy condition violating quantum contributions to the RSET could lead to significant deviations from classical collapse when a trapping horizon is on the verge of being formed.

Summary of the calculations

The previous two calculations point out that the building up of a significant RSET during the collapse, and therefore the appearance of quantum modifications to the collapse, depend in turn on the characteristics of the very collapse. If for whatever reason the collapse starts to significantly deviate from free fall, then, the building up of an energy-violating RSET could take over and further slow down the collapse. Surprisingly, the behaviour of the RSET is such that it allows two rather different scenarios to be conceptually self-consistent. One is the standard scenario: One assumes almost free-fall collapse and the RSET results in negligible values thus confirming the initial hypothesis. The other is the one presented here: One assumes significant deviations from free-fall and the RSET results in large values qualitatively consistent with the starting hypothesis.

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